Preface

The word "tears" is found in mathematical titles surprisingly often. One reads of "mathematics without tears," "geometry without tears," "topology without tears," "statistics without tears," and, of course, "calculus without tears," among others. Compare these juxtapositions of tears and mathematics with what the late Fields Medalist and member of Bourbaki, Laurent Schwartz, once famously wrote,"Il n'y a pas de mathematiques sans larmes . . . " [There is no mathematics without tears. . . .] It seems that there has been much weeping over mathematics, as well as disagreement over whether the weeping is necessary or not. Perhaps people have carried the tear metaphor too far, but the underlying sentiment behind tearlessness has also been long expressed in other ways too. Over the twentieth century we have had all manner of mathematics titles using words like, "outline," "nutshell," "simple," or "dummies."

For example, S. P. Thompson published *Calculus Made Easy* in 1910. It retains enough of a following today, more than a century later, that you can still buy fresh printings of the 1914 second edition on Amazon. Not even a more recent edition rewritten by Martin Gardener, no less, has been able to push the second edition into oblivion. Mathematics texts are like that. At the beginning of the twentieth century, some students were still taught from Euclid's *Elements*, in the original ancient Greek. In mathematics the basics last. This makes the modern mathematics instructor a bit nihilistic about choosing which textbook to use in teaching calculus. "Does it really matter which text I use?" they ask. At some level it indeed does not, so they just employ whatever text was used last year. "More of the same" becomes the standard operating procedure. But this cannot always hold true or we might be using Thompson today instead of any number of modern texts.

One reason more-of-the-same doesn't always hold is that the audience changes even when the basics don't. We explained in the seventh edition that modern students, with a lifelong exposure to modern graphical computer interfaces, cannot help but look at mathematics differently than previous generations. It is all too easy for them to have the impression that mathematics is an application of computer science, existing as a mere icon on desktops. Why learn mathematics, they reason, when it seems we can have it all at a "click?" Learning that computer science, not mathematics, is actually the application is a good first step. Learning that computers are not something to be believed, except in a conditional manner, is the definitive lesson for the computer age. That alone is reason enough to learn mathematics. The seventh edition dealt with this through a new thematic topic called "Numerical Monsters" (marked by in this new edition). These represent a form of *anti-numerical analysis*—"anti" because the topic aims to make computer errors as large as possible instead of minimizing them. They provide natural, self-contained mathematical applications that play off the finite representation of numbers within all computers. What is learned is fundamental, and qualitatively independent of code or platform. All of this was inconceivable to Euclid, Newton, or S. P. Thompson.

There is another reason why more-of-the-same is problematic. Much of the basic mathematics used in mathematically based fields was set about a century and half ago. At the time of Thompson, one hundred years ago, there was a gap between the application fields and the calculus exposition of that time. More-of-the-same has preserved that gap over all the subsequent years. When students move from calculus to the mathematics of one of these fields, they enter a strange world with mathematical customs that may even contradict what they have been taught in their calculus courses. For example, what are physics, chemistry, or engineering students supposed to make of the famous equation, $dE = \delta W + \delta Q$? It depicts a differential equalling the sum of two things that are not actually differentials. Puzzling to say the least. But what are these things on the right side? Students are told not to be alarmed in their field-specific texts because E is not actually a function of W or Q. Well, since δW and δQ are not actually differentials anyway, maybe that is okay then, or is it? Sometimes the δ's are replaced by d's with little bars through them to emphasize that some kind of unique-to-thermodynamics "mathematics" is in play. This representation is an anachronism dating from the nineteenth century, recalling dubious attempts to depict everything, in addition to functions, in terms of differentials alone. Calculus texts have simply not ventured to show how to proceed without the nineteenth century awkwardness still in play today.

This is not the only example of this phenomenon by any means. Such things have generated more confusion and "tears" than any mathematics course ever has. A good introductory applied calculus textbook ought to lead the reader to where the calculus properly connects to actual fields that calculus students may actually encounter in their subsequent training. This not only helps to stave off unnecessary "tears" but ultimately can lead to a more lucid standard of exposition for the fields in question. We described these connections with the thematic title "Gateway Applications" in the seventh edition. We have marked them by the symbol in the eighth edition. They should not be confused with "applications" that appear as tamed examples and staged problems typical in all textbooks. Instead, they take the reader from a calculus topic at hand directly to a mathematical tool often overlooked in calculus texts but crucial to an actual field, or they take the reader to an insight on how calculus sets the structure of an entire field, without actually pursuing the field. Now that's application!

In the seventh edition we introduced a number of these gateway applications, from Liapunov functions to thermodynamics and Legendre transformations. We also sketched out why these things are important and how they are actually used. In the eighth edition we have added a calculus-based explanation of entropy as a gateway application, showing how it naturally arises from simple calculus properties and how it fits both into statistical mechanics in physics and information theory. Gateway applications are not meant to replace those traditional "applications." They are meant to enhance the possibilities within a calculus course, either as source material for independent projects, or as enrichment for a course that an instructor may choose to explore to make a point to a class. Moreover, they are value-added when viewing the book as a future reference work. When students encounter the gaps between their calculus training and their subsequent courses, chances are they will find answers on how to bridge the gaps not available any place else. The eighth edition is no crib sheet to be discarded after the course is done.

There is yet one more reason why more-of-the-same is problematic. Over long enough timescales our best understanding of mathematics and how it is used does actually change. Unanswered questions linger even in conventional calculus that are not answered by more-of-the-same. These questions bother students, impeding their learning, and may even have bothered instructors when they were students. A relatively straightforward example is how we know that a minimization in a Lagrange multiplier problem really provides a minimum. Does the famous maximum entropy principle really provide a maximum? How does one know? An answer to this cannot be found in other mainstream calculus textbooks, oddly enough, but you can find it in the eighth edition. A subtler question on the minds of students is what is the difference between a differential of a function and the differential appearing in a multiple integral (i.e., think of $dy = f'(x)\,dx$ versus, say, $dV = dx\,dy\,dz$)? If you read Thompson, or nearly any other textbook, you may be forgiven for concluding that they are really just the same. But they are not the same. In fact, the differences turn out to have great significance, revealing structures that simplify and unify advanced calculus while having stunning implications for the fundamental differential equations governing many fields of science. All of this is found within the subject of *Exterior Calculus*.

Despite the power of exterior calculus it is not part of the normal more-of-the-same approach, at least in part because some of its development was after, or contemporary with, works like Thompson. But that still makes its current form nearly a hundred years old—just like yesterday from the perspective of mathematics pedagogy. Until now, a basic textbook-style treatment of the subject has been unavailable because it has been frozen out by more-of-the-same, causing it to be thought of as strictly an advanced topic. Technical monographs only reinforce this idea, but the structures while not low level, are not that high level in principle either. Vectors, linear algebra, basic calculus, some abstract imagination, and some clarifications about terminological customs is pretty much all that is needed. Maybe at some future date, the advanced calculus curriculum will be structured differently, but for now we offer an answer to the student's question about differentials in the form of the new Chapter 17 of the eighth edition. This is consistent with being innovative and living up to the subtitle: *A Complete Course*, while maintaining continuity and respecting tradition as much as is practical.

But, as with other value-added features of this volume, an instructor can simply ignore the material and teach a conventional program. There are few treatments of that traditional material more straightforward or succinct. But if the instructor wants to go a little further or a lot further; if the instructor wants to make a simple pedagogical point or identify a project for students; if the instructor just wants to point to a place where a student's questions can be answered, the eighth edition can help accomplish all of these things. Chapter 17 has been tried out in an advanced calculus course at the University of British Columbia, with positive results. But what is really intriguing to us is how many colleagues expressed interest in reading a textbook-style treatment of something that they always wanted to learn about themselves.

There is nothing wrong with books like *Calculus Made Easy*. We encourage readers to look at some, not only because alternative treatments can be helpful at times, but also to disabuse readers of any impression that they contain any sort of special educational magic beyond the usual protocol of diagrams, explanations, statements, derivations, definitions, worked examples, and exercises. There is also nothing wrong with modern interactive computer treatments of mathematics either. There are many advantages, such as hauling around software rather than paper, but even here the false allure of a mythical magic road to learning mathematics is tempting. Programmed learning still boils down to the same protocol as in a textbook: diagrams, explanations, etc. Add an instructor and answers in the back of the book, then you span the same pedagogical space. No matter how it is delivered, learning mathematics that is new to a student takes work and maybe enduring some hopefully temporary tears. There is no magic road. Therefore, instructors should call for fearlessness from students rather than worrying about their tearlessness. Challenge marks the path of greatest gain, particularly in a textbook where the path is fully known and clearly marked. Fearless rather than tearless students learn quickly what S. P. Thompson meant when he began his text, "What one fool can do, another can."

To the Student

You are holding what has become known as a "high-end" calculus text in the book trade. You are lucky. Think of it as having a high-end touring car instead of a compact economy car. But it is not high end in the material sense. It does not have scratch-and-sniff pages, sparkling radioactive ink, or anything else like that. It's the contents that set it apart. Unlike the car business, "high-end" book content is not priced any higher than any other books. It is one of the few consumer items where anyone can afford to buy into the high end. But there is a catch. Unlike cars, you have to do the work to achieve the promise of the book. So in that sense "high end" is more like a form of "secret" martial arts for your mind that the economy version cannot deliver. If you practise, your mind will become stronger. You will become more confident and disciplined. Secrets of the ages will become open to you. You will become fearless, as your mind longs to tackle any new mathematical challenge.

But hard work is the watchword. Practise, practise, practise. Think of how bees work busily to get their honey. There is a sort of "honey" in calculus. It is sweet when you finally get a new idea that you did not understand before. There are few experiences as great as figuring things out. That is one of the reasons why there has always been a booming world puzzle industry. In a high-end book there is more honey to be had than in recreational puzzles or lesser calculus texts. Doing exercises and checking against solutions in the back of the book are how you practise mathematics with a text. You can do essentially the same thing on a computer interface: you still do the problems and check the answers. However you do it, more exercises mean more practice and better performance.

There are numerous exercises in this text—too many for you to try them all perhaps, but be ambitious. Some are "drill" exercises to help you develop your skills in calculation. More important, however, are the problems that develop reasoning skills and your ability to apply the techniques you have learned to concrete situations. In some cases you will have to plan your way through a problem that requires several different "steps" before you can get to the answer. Other exercises are designed to extend the theory developed in the text and therefore enhance your understanding of the concepts of calculus. Think of the problems as a tool to help you correctly wire your mind. You may have a lot of great components in your head, but if you don't wire the components together properly, your "home theatre" won't work.

The exercises vary greatly in difficulty. Usually, the more difficult ones occur toward the end of exercise sets, but these sets are not strictly graded in this way because exercises on a specific topic tend to be grouped together. Also, "difficulty" can be subjective. For some students, exercises designated difficult may seem easy, while exercises designated easy may seem difficult. Nonetheless, some exercises in the regular sets are marked with the symbols !, which indicates that the exercise is somewhat more difficult than most, or ?, which indicates a more theoretical exercise. The theoretical ones need not be difficult; sometimes they are quite easy. Most of the problems in the *Challenging Problems* section forming part of the *Chapter Review* at the end of most chapters are also on the difficult side.

It is not a bad idea to review the background material in Chapter P (Preliminaries), even if your instructor does not refer to it in class.

If you find some of the concepts in the book difficult to understand, *re-read* the material slowly, if necessary several times; *think about it*; formulate questions to ask fellow students, your TA, or your instructor. Don't delay. It is important to resolve your problems as soon as possible. If you don't understand today's topic, you may not understand how it applies to tomorrow's either. Mathematics builds from one idea to the next. Testing your understanding of the later topics also tests your understanding of the earlier ones. Do not be discouraged if you can't do *all* the exercises. Some are very difficult indeed. The range of exercises ensures that nearly all students can find a comfortable level to practise at, while allowing for greater challenges as skill grows.

Answers for most of the odd-numbered exercises are provided at the back of the book. Exceptions are exercises that don't have short answers: for example, "Prove that ..." or "Show that ..." problems where the answer is the whole solution. A *Student Solutions Manual* that contains detailed solutions to even-numbered exercises is available.

Besides ! and ? used to mark more difficult and theoretical problems, the following symbols are used to mark exercises of special types:

- Exercises pertaining to differential equations and initial-value problems. (It is not used in sections that are wholly concerned with DEs.)
- Problems requiring the use of a calculator. Often a scientific calculator is needed. Some such problems may require a programmable calculator.
- Problems requiring the use of either a graphing calculator or mathematical graphing software on a personal computer.
- Problems requiring the use of a computer. Typically, these will require either computer algebra software (e.g., Maple, Mathematica) or a spreadsheet program such as Microsoft Excel.

To the Instructor

This book covers the material usually encountered in a three- or four-semester real-variable calculus program, involving real-valued functions of a single real variable (differential calculus in Chapters 1–4 and integral calculus in Chapters 5–8), as well as vector-valued functions of a single real variable (covered in Chapter 11), real-valued functions of several real variables (in Chapters 12–14), and vector-valued functions of several real variables (in Chapters 15 and 16). Chapter 9 concerns sequences and series, and its position is rather arbitrary.

Chapter 10 contains necessary background on vectors and geometry in 3-space as well as a bit of linear algebra that is useful, although not absolutely essential, for the understanding of subsequent multivariable material. Most of the material requires only a reasonable background in high school algebra and analytic geometry. (See Chapter P—Preliminaries for a review of this material.) However, some optional material is more subtle and/or theoretical and is intended for stronger students, special topics, and reference purposes. It also allows instructors considerable flexibility in making points, answering questions, and selective enrichment of a course. Chapter 18, for example, is a compact treatment of linear ordinary differential equations which may provide supplementary material or become a major topic in a multi-topic course.

Changes in the eighth edition include numerous improvements and clarifications throughout, including notational adjustments and corrections. Major additions include Taylor's formula in terms of functions of n variables (Section 12.9); the classification of extrema for functions with constraints (Section 13.4); the gateway application, entropy in statistical mechanics, and information theory (Section 13.9); and Chapter 17, "Differential Forms and Exterior Calculus."

There is a wealth of material here—too much to include in any course. It was never intended to be otherwise. You must select what material to include and what to omit, taking into account the background and needs of your students. At the University of British Columbia, where one author taught for 34 years, and at the University of Western Ontario, where the other author continues to teach, calculus is divided into four semesters, the first two covering single-variable calculus, the third covering functions of several variables, and the fourth covering vector calculus. In none of these courses was there enough time to cover all the material in the appropriate chapters; some sections are always omitted. The text is designed to allow students and instructors to conveniently find their own level while enhancing any course from general calculus to courses focused on science and engineering students.

Several supplements are available for use with *Calculus: A Complete Course, 8th Edition*. Available to students is the **Student Solutions Manual** (ISBN: 9780321862938): This manual contains detailed solutions to all the even-numbered exercises, prepared by the authors. There are also such Manuals for the split volumes, *Single Variable Calculus* (ISBN: 9780321877468), and *Calculus of Several Variables* (ISBN: 9780321877475).

Available to instructors is an **Instructor's Resource CD-ROM** (IRCD), (ISBN: 9780321874733), including the following:

- **Instructor's Solutions Manual**,
- **Text Solutions in online-publishable form**,
- **Pearson TestGen**. TestGen is testing software that enables instructors to view and edit the existing questions, (over 1,500 test questions are provided), and add questions, generate tests, and distribute the tests in a variety of formats.
- **Image Library**, which contains all of the figures in the text provided as individual enlarged .pdf files suitable for printing to transparencies.

These supplements are available for download from a password-protected section of Pearson Education Canada's online catalogue (catalogue.pearsoned.ca). Navigate to this book's catalogue page to view a list of those supplements that are available. See your local sales representative for details and access.

Also available to qualified instructors are **MyMathLab**® and **MathXL**® Online Courses for which access codes are required.

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The expunging of errors and obscurities in a text is an ongoing and asymptotic process; hopefully each edition is better than the previous one. Nevertheless, some such imperfections always remain, and we will be grateful to any readers who call them to our attention, or give us other suggestions for future improvements.

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