FOURTH CANADIAN EDITION

Elementary and Middle School Mathematics

Teaching Developmentally

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PEARSON

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Original edition published by Pearson Education, Inc., Upper Saddle River, New Jersey, USA.

10 9 8 7 6 5 4 3 2 1 [CKV]

Library and Archives Canada Cataloguing in Publication

Van de Walle, John A., author
Elementary and middle school mathematics: teaching developmentally / John A. Van de Walle, Late of Virginia Commonwealth University, Karen S. Karp, University of Louisville, Jennifer M. Bay-Williams, University of Louisville, Lynn M. McGarvey, University of Alberta, Sandra Folk, University of Toronto.—Fourth Canadian edition.

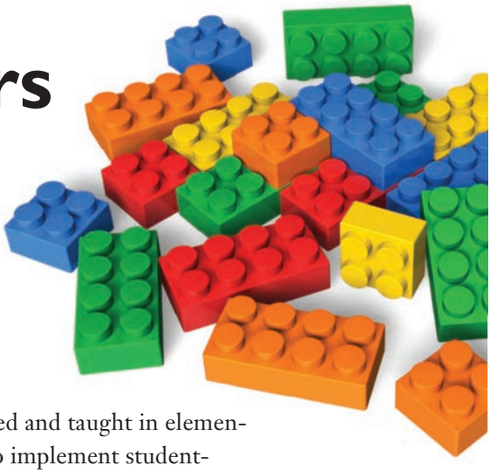
Revision of: Elementary and middle school mathematics: teaching
developmentally / John A. Van de Walle ... [et al.].—3rd Canadian ed.—Toronto: Pearson Allyn & Bacon, c2011.
Includes bibliographical references and index. ISBN 978-0-205-87195-7 (pbk.)
1. Mathematics—Study and teaching (Elementary). 2. Mathematics—Study and teaching (Middle school). I. Karp, Karen S., author II. Bay-Williams, Jennifer M., author III. McGarvey, Lynn M., author IV. Folk, Sandra, author V. Title.

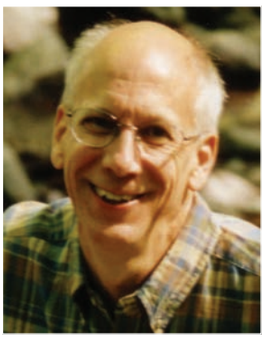
QA135.6.V36 2014 372.7'044 C2013-906895-3

PEARSON

ISBN 978-0-20-587195-7

About the Authors





John A. Van de Walle was a professor emeritus at Virginia Commonwealth University. He was a mathematics education consultant who regularly gave professional development workshops for K–8 teachers in the United States and Canada. He visited and taught in elementary school classrooms and worked with teachers to implement student-centered math lessons. He co-authored the Scott Foresman-Addison Wesley Mathematics K–6 series and contributed to the Pearson School mathematics program enVisionMATH. Additionally, he wrote numerous chapters and articles for the National Council of Teachers of Mathematics (NCTM) books and journals and was very active in NCTM, including serving on the board of directors, chairing the educational materials committee, and speaking at national and regional meetings.

Karen S. Karp is a professor of mathematics education at the University of Louisville (Kentucky). Prior to entering the field of teacher education she was an elementary school teacher in New York. Karen is a co-author of *Feisty Females: Inspiring Girls to Think Mathematically*, which is aligned with her research interests on teaching mathematics to diverse populations. With Jennifer, Karen co-edited *Growing Professionally: Readings from NCTM Publications for Grades K–8* and co-authored (along with Janet Caldwell) *Developing Essential Understanding of Addition and Subtraction for Teaching Mathematics in Pre-K–Grade 2*. She is a former member of the board of directors of the National Council of Teachers of Mathematics (NCTM) and a former president of the Association of Mathematics Teacher Educators (AMTE). She continues to work in classrooms with elementary and middle school teachers and with teachers at all levels who work with students with disabilities.

Jennifer M. Bay-Williams is a professor of mathematics education at the University of Louisville (Kentucky). Jennifer has published many articles on teaching and learning in NCTM journals. She has also co-authored numerous books, including *Developing Essential Understanding of Addition and Subtraction for Teaching Mathematics in Pre-K–Grade 2*, *Math and Literature: Grades 6–8*, *Math and Nonfiction: Grades 6–8*, and *Navigating Through Connections in Grades 6–8*. Jennifer taught elementary, middle, and high school in Missouri and in Peru, and continues to work in classrooms at all levels with students and with teachers. Jennifer is on the board of directors for TODOS: Equity for All, is the editor for the 2012 NCTM Yearbook, and is a former president of the Association of Mathematics Teacher Educators (AMTE).

About the Contributor

Jonathan Wray is the technology contributor to *Elementary and Middle School Mathematics: Teaching Developmentally*. He is the instructional facilitator for secondary mathematics curricular programs in the Howard County public school system. He is the president elect of the Association of Maryland Mathematics Teacher Educators (AMMTE) and past president of the Maryland Council of Teachers of Mathematics (MCTM). He has been recognized for his expertise in infusing technology in mathematics teaching, receiving the Outstanding Technology Leader in Education award for his school district from the Maryland Society for Educational Technology (MSET). Jon is also actively engaged in NCTM, serving on the editorial panels of *Teaching Children Mathematics* and ON-Math. He has served as a primary and intermediate grades classroom teacher, gifted/talented resource teacher, elementary mathematics specialist, curriculum and assessment developer, grant project manager, and educational consultant.

About the Canadian Author

Lynn M. McGarvey is an associate professor of mathematics education at the University of Alberta. Lynn's research focuses on the mathematical reasoning of young children. She has written many articles for research and professional audiences, particularly on the topics of algebraic thinking, spatial reasoning, and patterning in the early years. She is a long-time member of NCTM and has served on a number of task forces and committees, including as editorial panel chair for *Teaching Children Mathematics*. She is a former junior high school teacher who now spends a considerable amount of time working with children and teachers in preschools and kindergartens. Lynn has taught mathematics curriculum and pedagogy courses to thousands of pre-service and in-service elementary teachers and has won multiple teaching awards for this work.

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SECTION II

Development of Mathematical Concepts and Procedures

This section serves as the application of the core ideas of Section I. Here you will find chapters on every major content area in the pre-K–8 mathematics curriculum. Numerous inquiry-based tasks and problems to engage students are interwoven with a discussion of the mathematical content and how children develop their understanding of that content. At the outset of each chapter, you will find a listing of "Big Ideas," the mathematical umbrella for the chapter. Also included are ideas for incorporating children's literature, technology, and assessment. These chapters are designed to help you develop pedagogical strategies and to serve as a resource for your teaching now and in the future.

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CHAPTER 12

Preface

New to This Edition

The fourth Canadian edition has been revised to include the following changes to better prepare teachers to teach mathematics to Canadian learners:

- **Section I (Chapters 1–7)** has been substantially revised to emphasize and describe mathematics teaching, learning, and curriculum across Canada. In particular, twenty-first century knowledge and skills including problem solving, creativity, collaboration, communication, and social responsibility are introduced.
- In the third edition, Chapter 1 was based substantially on two aspects: NCTM *Principles and Standards for School Mathematics* (NCTM, 2000), and provincial, national, and international testing programs. **Chapter 1 now examines the curriculum documents of the provinces and territories across Canada** and notes their similarities and differences in terms of goals for student learning, mathematics content, and mathematical processes.
- **Diverse learners** are also introduced in **Chapter 1** as diversity is the norm in most Canadian classrooms. Diverse learning needs are described here and throughout the text as opportunities to enhance learning for all students rather than as something to eliminate. The chapter explicitly addresses First Nations, Métis, and Inuit students, English language learners, and learners with special needs including those with cognitive, behavioural, and physical exceptionalities.
- **Chapter 2 is substantially revised to expand on the nature of mathematics**—from being the science of patterns or as problem solving to a more encompassing perspective that mathematics is *a way of thinking and seeing*. The tasks within all the chapters are revised to reflect this expanded viewpoint. Canadian examples are used throughout. **The Chapter 2 section on learning theory is also revised**. Rather than singling out specific learning theories, the emphasis is on helping teachers recognize widespread beliefs about learning and practices reflected in the metaphors we use (e.g., "I'm falling behind").
- The first two chapters set the stage for revisions in **Chapter 3**. Its previous focus on teaching through problem solving now takes **a broader focus on mathematical inquiry through rich tasks and classroom discourse**. A more in-depth discussion is provided that outlines the features of a rich task using a multitude of examples. The discussion surrounding tasks such as drill and homework have been incorporated into **Chapter 3**. Problem solving is considered one form of inquiry alongside other types of worthwhile tasks. Given this shift in focus, *mathematical habits of mind* replace problem solving strategies to encompass broader perspectives of mathematical thinking and doing.
- In previous editions, lesson preparation and planning based on a three-phase lesson format was been spread across two chapters. In this fourth Canadian edition, **preparation and planning** are placed together in **Chapter 4**, entitled "Preparing to Teach and Planning for Mathematics Learning."
- **Chapter 5** has been revised to use the framework of assessment *for* learning, *as* learning, and *of* learning throughout, using the questions: For what purpose? Who is responsible? What is assessed? What tools are used? and How is assessment data recorded? **Substantial attention is paid to assessment** of content, processes, and dispositions within Canadian

curriculum documents and in this chapter. Many more examples of children's work and assessment tools have been added to this chapter.

- **Chapter 6**, now Teaching Mathematics for All Learners, has also been substantially revised. In the previous edition, the chapter focused extensively on specific disabilities and general ways to address specific learning needs. The current chapter has been revised to **focus on differentiated teaching and learning**; in particular, it builds on criteria for rich tasks from Chapter 3 to create tiered tasks, learning centres, choice boards, and so on. **Diversity within the classroom** is also addressed in more detail and takes into account different cultural ways of learning, culturally responsive teaching strategies, and linguistic diversity. Students with special needs are addressed in more detail through the framework of Response to Intervention, which offers a way to consider a tiered student support system and forms of intervention.
- The focus for **Chapter 7**, which was previously on technology and school mathematics, has been **expanded to address Tools for Learning Mathematics**, of which technology is one type. The chapter is based on three types of tools used in Canadian curriculum, including (1) concrete materials and physical models (e.g., manipulatives); (2) visual and graphic representations (e.g., pictures, diagrams, schematic drawings, graphs); and (3) technology-based tools (e.g., calculators, dynamic geometry software). The chapter provides details on how tools can be used appropriately in the classroom to enhance learning.

Section II of the Textbook has been substantially updated with new research, along with a number of new features.

- **New adaptations and accommodations for diverse learners**, including English language learners and students with difficulties in mathematics, appear not only in the narrative in Section I but also in many activities through direct examples and descriptions for the various content areas in Section II. The increased emphasis on diversity will be obvious to those who have used the book in the past.
- **New samples of authentic student work** illustrate student thinking. Student responses present glimpses into how students think about problems and what students' written work on mathematical tasks looks like, increasing teachers' awareness of how rich students' mathematical thinking can be—and how high our expectations should be.
- **Increased early childhood coverage** provides expanded emphasis on and reorganization of early numeracy in **Chapters 8 and 9**. Based on learning for the areas of number, relations, and operations, the work with early learners is seen as the essential foundation for number sense and mathematical ways of thinking.
- **New Formative Assessment Notes** in each chapter in Section II guide readers through ideas they can test with individual students or students in groups. Formative assessment is one of the key tools in finding out what students are thinking, and thereby identifying their areas of strength and weakness. **Chapter 5** contains a more detailed description of formative assessments organized in three major assessment areas: tasks, observation, and interviews. To bring these ideas to life and to make them more directly linked to the content, these Formative Assessment Notes are included throughout the content area chapters to support teachers in the effective use of formative assessment, which is directly connected to increased student achievement.
- **Extensively updated information on how to effectively integrate new technological tools** to support teaching and learning appears in select Activities throughout the text.
- **A reorganization of Chapters 12 and 13** emphasizes both strategies for computation and estimation for addition and subtraction in Chapter 12, and the same for

multiplication and division in Chapter 13. This is a change from the third edition, which separated developing strategies for whole number computation and estimation for the four operations.

- **A discussion on engaging families in meaningful ways** to help students learn mathematics appears in **Chapter 4**.
- **Additional attention to classroom discourse** now appears in **Chapter 3**, Mathematical Inquiry through Rich Tasks and Classroom Discourse. The coverage includes how to conduct productive discussion sessions and develop effective questioning, and is illustrated with a vignette.

Other Changes of Note

Much has changed on the landscape of mathematics education, and so many aspects of the book have been updated to reflect those changes. In addition to the changes listed above, the following substantive changes have been made:

- There is an increased focus on the research-based developmental model of developing basic facts, and new activities to support basic fact mastery appear in Chapter 10.
- The content on algebraic thinking has been adapted to align with current research and changes to curricula across Canada. There is an increased emphasis on equivalence and variables, including adding the number-line representation of variables and making the properties more explicit.
- Chapter 15 (Developing Fraction Concepts) has greatly expanded sections on partitioning and on equivalence to reflect three recent research reviews that have indicated that this is essential to all advanced fraction work and success in algebra.
- Chapter 16 (Developing Strategies for Fraction Computation) now includes Activities—10 new ideas for developing understanding of fraction operations.
- Chapter 18 has been shortened, had new activities added, and been refocused to address understanding of ratios more deeply (with less focus on connecting to other content areas).
- The chapter on measurement, Chapter 19, has been reorganized. Previously, the development of all measurement formulas was shared at the end of the chapter; now, the formulas are integrated with the corresponding measurement topic (e.g., area or volume). The content has also been revised to ensure the focus is on metric measurement.
- Chapter 21 gives more explicit attention to distinguishing between numerical data and categorical data.
- Chapter 23 includes a significantly revised section on order of operations and numerous new activities.

What You Will Find in This Book

When you look at the table of contents, you will see that the chapters are separated into two distinct sections. The first section, consisting of seven chapters, deals with important ideas that cross the boundaries of specific areas of content. The second section, consisting of 16 chapters, offers teaching suggestions and activities for every major mathematics topic in the pre-kindergarten to Grade 8 curriculum. Chapters in Section I offer perspectives on the challenging task of helping students learn mathematics. Having a feel for the discipline of mathematics—that is, to know what it means to "do mathematics"—is critical to learning how to teach mathematics well. In addition, understanding perspectives on learning

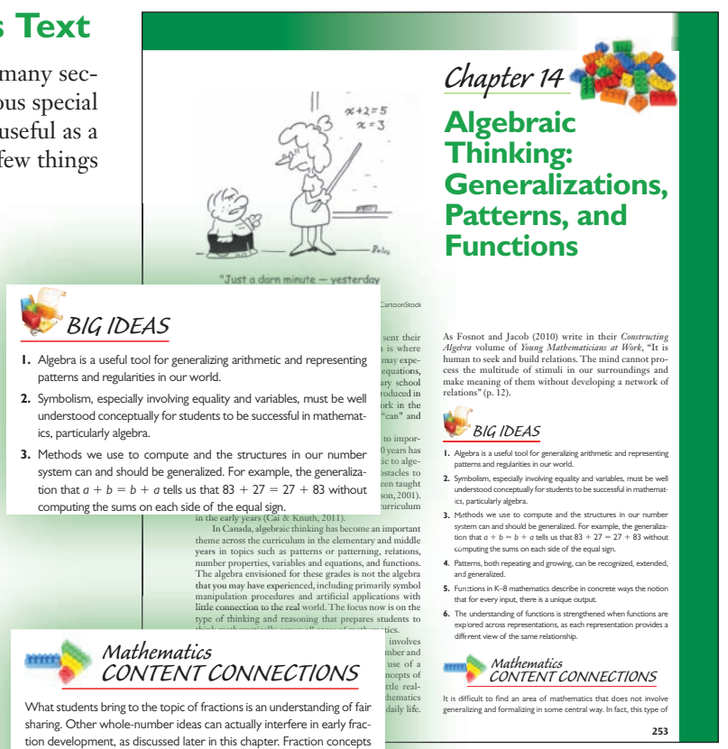
mathematics and how different perspectives reflect different approaches to teaching provides a foundation and rationale for how to teach and assess pre-K–8 students.

Importantly, you will be teaching diverse students, including students who are English language learners, are gifted, and/or have difficulties learning. You will learn how to apply instructional strategies in ways that support and challenge all learners. Formative assessment strategies, strategies for diverse learners, and effective use of learning tools are addressed in specific chapters in Section I (Chapters 5, 6, and 7, respectively), and throughout the Section II chapters.

Each chapter of Section II focuses on one of the major content areas in pre-K–8 mathematics curriculum. It begins with identifying the big ideas for that content, then provides guidance on how students best learn that content, along with many worthwhile tasks to engage students in understanding mathematics. Reflecting on the activities as you read can help you think about the mathematics from the perspective of the student. As often as possible, take out pencil and paper and try the problems so that you actively engage in your learning about students learning mathematics. We hope this book will increase your own understanding of mathematics, of the students you teach, and of how to teach them well.

Some Special Features of this Text

Upon flipping through the book, you will notice many section headings, a large number of figures, and various special features. All are designed to make the book more useful as a textbook and as a long-term resource. Here are a few things to look for.



Big Ideas ▶

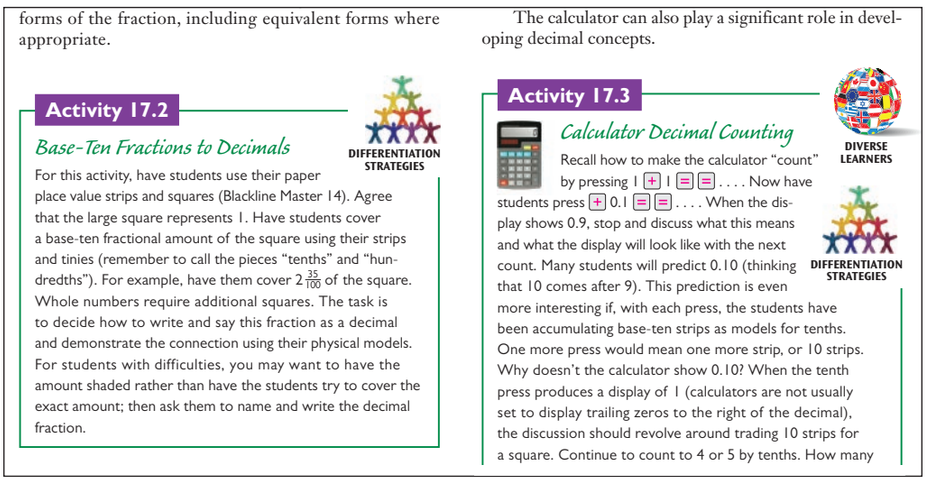
Much of the current research and literature in mathematics education suggests that teachers plan their instruction around "big ideas" rather than isolated skills or concepts. At the beginning of each chapter in Section II, you will find a list of the key mathematical ideas associated with the chapter. Teachers find these lists helpful for quickly getting a picture of the mathematics they are teaching.

Mathematics Content Connections ▶

Following the Big Ideas lists are brief descriptions of other content areas in mathematics that are related to the content of the current chapter. These lists are offered to help you be more aware of the potential interaction of content as you plan lessons, diagnose students' difficulties, and learn more yourself about the mathematics you are teaching.

Activities ▶

The numerous activities found in every chapter of Section II have always been rated by readers as one of the most valuable parts of the book. Some activity ideas are described directly in the text and in the illustrations. Others are presented in the numbered Activity boxes. Every activity is a problem-based task (as described in Chapter 3) and is designed to engage students in doing mathematics. New adaptations and accommodations for diverse learners and differentiation strategies for students with difficulties are included in many activities.



forms of the fraction, including equivalent forms where appropriate.

Activity 17.2

DIFFERENTIATION STRATEGIES

Base-Ten Fractions to Decimals

For this activity, have students use their paper place value strips and squares (Blackline Master 14). Agree that the large square represents 1. Have students cover a base-ten fractional amount of the square using their strips and tinies (remember to call the pieces "tenths" and "hundredths"). For example, have them cover $2\frac{35}{100}$ of the square. Whole numbers require additional squares. The task is to decide how to write and say this fraction as a decimal and demonstrate the connection using their physical models. For students with difficulties, you may want to have the amount shaded rather than have the students try to cover the exact amount; then ask them to name and write the decimal fraction.

The calculator can also play a significant role in developing decimal concepts.

Activity 17.3

DIVERSE LEARNERS

DIFFERENTIATION STRATEGIES

Calculator Decimal Counting

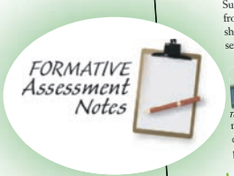
Recall how to make the calculator "count" by pressing 1 [+] 1 [=] [=] . . . . Now have students press [+] 0.1 [=] [=] . . . . When the display shows 0.9, stop and discuss what this means and what the display will look like with the next count. Many students will predict 0.10 (thinking that 10 comes after 9). This prediction is even more interesting if, with each press, the students have been accumulating base-ten strips as models for tenths. One more press would mean one more strip, or 10 strips. Why doesn't the calculator show 0.10? When the tenth press produces a display of 1 (calculators are not usually set to display trailing zeros to the right of the decimal), the discussion should revolve around trading 10 strips for a square. Continue to count to 4 or 5 by tenths. How many

Technology Ideas ▶

Technology is an important tool for learning mathematics, as you will learn in Chapter 7. We have infused technology ideas throughout Section II. An icon is used to identify those places within the text or an activity where a technology idea or resource is discussed. Descriptions include open-source (free) software, applets, and other Web-based resources, as well as calculator ideas.

Formative Assessment Notes ▶

Assessment should be an integral part of instruction; so, it makes sense to think about what to be listening for (assessing) as you read about different areas of content development. Throughout the content chapters, you will see Formative Assessment Note icons indicating a short description of ways to assess the topic in that section. Reading these assessment notes as you read the text can help you understand how best to help your students.



Data Analysis: Measures of Centre and Variability 443

0 1 2 3 4 5 6 7 8 9 10 11 12

FIGURE 21.17 Move data points in toward the centre or balance point without changing the balance around that point. When you have all points at the same value, that is the balance or the *mean*.

Changes in the Mean. The balance approach to finding the mean clearly illustrates that different data distributions can have the same mean. Especially for small sets of data, the mean is significantly affected by extreme values. For example, suppose another toy with a price of $20 is added to the six we have been using in the examples. How will the mean change? If the $1 toy were removed, how would the mean be affected? Suppose that one new toy is added that increases the mean from $6 to $7. How much does the new toy cost? Students should be challenged with questions such as these using small sets of data and either the balance or the levelling concept.

TECHNOLOGY

The NCTM e-Examples, Applet 6.6, "Comparing Properties of the Mean and the Median," shows seven data points that can be dragged back and forth along a number line with the mean and median updated instantly. The applet allows students to see how stable the median is and how changing one point can affect the mean.

FORMATIVE Assessment Notes

Consider using a **diagnostic interview** to assess whether students are able to determine the best measure of centre to use in a given situation, such as the average height of students in the class. You can begin with general questions such as these: "What is an average? What does the mean represent? What does the median represent? What is the difference between the mean and the median? What is each useful for?" Then move to more analytical questions: "Which should we use for this set of data? Might we use a different measure of centre in another class? When you've found the average height of the students in our class, is it possible that no one is that height? Why?" ■

Variability

Measures of variability also need explicit attention (Franklin & Garfield, 2006; Franklin et al., 2005; Rossman, Chance, & Medina, 2006; Scheaffer, 2006). Students often do not have a clear understanding of variability because the time spent analyzing measure of centre dominates the data analysis portion of the teacher's long-range plan. Shaughnessy (2006) summarized the findings on what students should know about variability in the following list, starting with basic notions and progressing to more sophisticated ideas:

1. attending to outliers or extremes (but not necessarily on the full distribution of the data)
2. considering change over time (which can lead into discussions of other types of variation)
3. examining variability as the full range of data (Range is everything that occurs, but it doesn't reveal the frequency of different events within the range.)
4. considering variability as the likely range or expected value
5. looking at how far data points are from the centre (e.g., the mean)
6. looking at how far off a set of data is from some fixed value

In order to be prepared to teach students variability beyond outliers and extremes, it is important to be aware of the sources of variability that occur in statistics. Franklin and colleagues (2005) suggest that students focus on increasingly sophisticated sources of variability, starting with variability within a group (e.g., the varying lengths of students' names, varying family sizes, and so on). When students create a bar graph of class data and compare the data collected, they are discussing the variability within a group.

Next is variability within and between groups. Students might compare the variability of favourite music choices of Grade 5 students compared to students in Grade 8. In addition, middle school students study how the change in one variable relates to change in another variable—yes, algebra! Students analyze two variables to see whether there is a relationship (as discussed earlier in the section on scatter plots). Students also explore sampling variability (Franklin et al., 2005). When students flip a coin 10 times as a sample, they may get 5 heads and 5 tails, but they also may get many other results (even 0 heads and 10 tails). This is sampling variability. The larger the sample, the more the data reflect the expected values (50 percent heads, 50 percent tails).

Lastly, students can examine *natural* and *induced* variability. For example, plants grow at different rates. When one flower naturally grows taller than the one right next to it in the garden, that is natural variability. If the two plants were in two different gardens, then other variables come into play: fertilization, amount of sunlight, amount of water, and so on, which can "induce" different growth rates. Knowing these variability terms is less important than knowing that in designing an experiment, we must look at one factor (e.g., sunlight) and all other factors should be

End-of-Chapter Resources ▶

The end of each chapter includes two major subsections: *Resources*, which includes "Literature Connections" (found in all Section II chapters), "Recommended Readings," and "Online Resources"; and *Reflections*, which includes "Writing to Learn" and "For Discussion and Exploration."

Literature Connections

Section II chapters contain examples of great children's literature for launching into the mathematics concepts in the chapter just read.

Recommended Readings

In this section, you will find an annotated list of articles and books to augment the information found in the chapter.

Online Resources

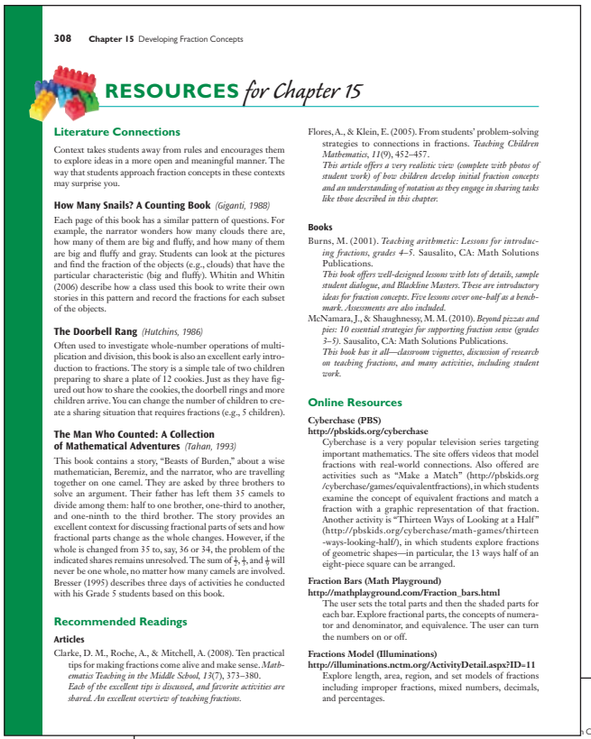
At the end of each chapter, you will find an annotated list of some of the best Web-based resources along with their website addresses so that you can further explore how to infuse technological tools into instruction to support student learning.

Writing to Learn

Questions are provided that help you reflect on the important pedagogical ideas related to the content in the chapter.

For Discussion and Exploration

These questions ask you to explore an issue related to that chapter's content, applying what you have learned.



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RESOURCES *for Chapter 15*

Literature Connections

Context takes students away from rules and encourages them to explore ideas in a more open and meaningful manner. The way that students approach fraction concepts in these contexts may surprise you.

How Many Snails? A Counting Book *(Giganti, 1988)*

Each page of this book has a similar pattern of questions. For example, the narrator wonders how many clouds there are, how many of them are big and fluffy, and how many of them are big and fluffy and gray. Students can look at the pictures and find the fraction of the objects (e.g., clouds) that have the particular characteristic (big and fluffy). Whitin and Whitin (2006) describe how a class used this book to write their own stories in this pattern and record the fractions for each subset of the objects.

The Doorbell Rang *(Hutchins, 1986)*

Often used to investigate whole-number operations of multiplication and division, this book is also an excellent early introduction to fractions. The story is a simple tale of two children preparing to share a plate of 12 cookies. Just as they have figured out how to share the cookies, the doorbell rings and more children arrive. You can change the number of children to create a sharing situation that requires fractions (e.g., 5 children).

The Man Who Counted: A Collection of Mathematical Adventures *(Tahan, 1993)*

This book contains a story, "Beasts of Burden," about a wise mathematician, Beremiz, and the narrator, who are travelling together on one camel. They are asked by three brothers to solve an argument. Their father has left them 35 camels to divide among them: half to one brother, one-third to another, and one-ninth to the third brother. The story provides an excellent context for discussing fractional parts of sets and how fractional parts change as the whole changes. However, if the whole is changed from 35 to, say, 36 or 34, the problem of the indicated shares remains unresolved. The sum of $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{9}$ will never be one whole, no matter how many camels are involved. Bresser (1995) describes three days of activities he conducted with his Grade 5 students based on this book.

Recommended Readings

Articles

Clarke, D. M., Roche, A., & Mitchell, A. (2008). Ten practical tips for making fractions come alive and make sense. *Mathematics Teaching in the Middle School, 13*(7), 373–380.
Each of the excellent tips is discussed, and favorite activities are shared. An excellent overview of teaching fractions.

Flores, A., & Klein, E. (2005). From students' problem-solving strategies to connections in fractions. *Teaching Children Mathematics, 11*(9), 452–457.
This article offers a very realistic view (complete with photos of student work) of how children develop initial fraction concepts and an understanding of notation as they engage in sharing tasks like those described in this chapter.

Books

Burns, M. (2001). *Teaching arithmetic: Lessons for introducing fractions, grades 4–5.* Sausalito, CA: Math Solutions Publications.
This book offers well-designed lessons with lots of details, sample student dialogue, and Blackline Masters. These are introductory ideas for fraction concepts. Five lessons cover one-half as a benchmark. Assessments are also included.

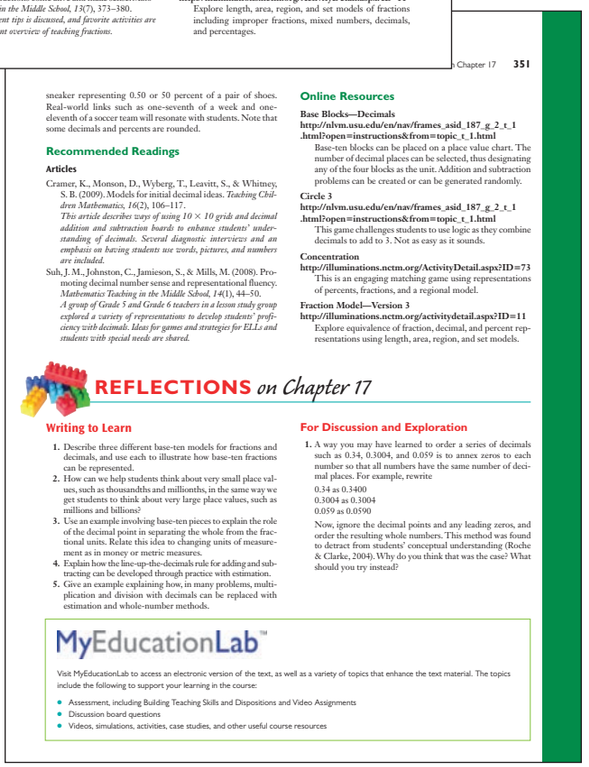
McNamara, J., & Shaughnessy, M. M. (2010). *Beyond pizzas and pies: 10 essential strategies for supporting fraction sense (grades 3–5).* Sausalito, CA: Math Solutions Publications.
This book has it all—classroom vignettes, discussion of research on teaching fractions, and many activities, including student work.

Online Resources

Cyberchase (PBS)
http://pbskids.org/cyberchase
Cyberchase is a very popular television series targeting important mathematics. The site offers videos that model fractions with real-world connections. Also offered are activities such as "Make a Match" (http://pbskids.org/cyberchase/games/equivalentfractions), in which students examine the concept of equivalent fractions and match a fraction with a graphic representation of that fraction. Another activity is "Thirteen Ways of Looking at a Half" (http://pbskids.org/cyberchase/math-games/thirteen-ways-looking-half/), in which students explore fractions of geometric shapes—in particular, the 13 ways half of an eight-piece square can be arranged.

Fraction Bars (Math Playground)
http://mathplayground.com/Fraction_bars.html
The user sets the total parts and then the shaded parts for each bar. Explore fractional parts, the concepts of numerator and denominator, and equivalence. The user can turn the numbers on or off.

Fractions Model (Illuminations)
http://illuminations.nctm.org/ActivityDetail.aspx?ID=11
Explore length, area, region, and set models of fractions including improper fractions, mixed numbers, decimals, and percentages.



Chapter 17 **351**

sneaker representing 0.50 or 50 percent of a pair of shoes. Real-world links such as one-seventh of a week and one-eleventh of a soccer team will resonate with students. Note that some decimals and percents are rounded.

Recommended Readings

Articles

Cramer, K., Monson, D., Wyberg, T., Leavitt, S., & Whitney, S. B. (2009). Models for initial decimal ideas. *Teaching Children Mathematics, 16*(2), 106–117.
This article describes ways of using 10 × 10 grids and decimal addition and subtraction boards to enhance students' understanding of decimals. Several diagnostic interviews and an emphasis on having students use words, pictures, and numbers are included.

Suh, J. M., Johnston, C., Jamieson, S., & Mills, M. (2008). Promoting decimal number sense and representational fluency. *Mathematics Teaching in the Middle School, 14*(1), 44–50.
A group of Grade 5 and Grade 6 teachers in a lesson study group explored a variety of representations to develop students' proficiency with decimals. Ideas for games and strategies for ELLs and students with special needs are shared.

Online Resources

Base Blocks—Decimals
http://nlvm.usu.edu/en/nav/frames_asid_187_g_2_t_1.html?open=instructions&from=topic_t_1.html
Base-ten blocks can be placed on a place value chart. The number of decimal places can be selected, thus designating any of the four blocks as the unit. Addition and subtraction problems can be created or can be generated randomly.

Circle 3
http://nlvm.usu.edu/en/nav/frames_asid_187_g_2_t_1.html?open=instructions&from=topic_t_1.html
This game challenges students to use logic as they combine decimals to add to 3. Not as easy as it sounds.

Concentration
http://illuminations.nctm.org/ActivityDetail.aspx?ID=73
This is an engaging matching game using representations of percents, fractions, and a regional model.

Fraction Model—Version 3
http://illuminations.nctm.org/activitydetail.aspx?ID=11
Explore equivalence of fraction, decimal, and percent representations using length, area, region, and set models.

REFLECTIONS *on Chapter 17*

Writing to Learn

1. Describe three different base-ten models for fractions and decimals, and use each to illustrate how base-ten fractions can be represented.
2. How can we help students think about very small place values, such as thousandths and millionths, in the same way we get students to think about very large place values, such as millions and billions?
3. Use an example involving base-ten pieces to explain the role of the decimal point in separating the whole from the fractional units. Relate this idea to changing units of measurement as in money or metric measures.
4. Explain how the line-up-the-decimals rule for adding and subtracting can be developed through practice with estimation.
5. Give an example explaining how, in many problems, multiplication and division with decimals can be replaced with estimation and whole-number methods.

For Discussion and Exploration

1. A way you may have learned to order a series of decimals such as 0.34, 0.3004, and 0.059 is to annex zeros to each number so that all numbers have the same number of decimal places. For example, rewrite
0.34 as 0.3400
0.3004 as 0.3004
0.059 as 0.0590
Now, ignore the decimal points and any leading zeros, and order the resulting whole numbers. This method was found to detract from students' conceptual understanding (Roche & Clarke, 2004). Why do you think that was the case? What should you try instead?

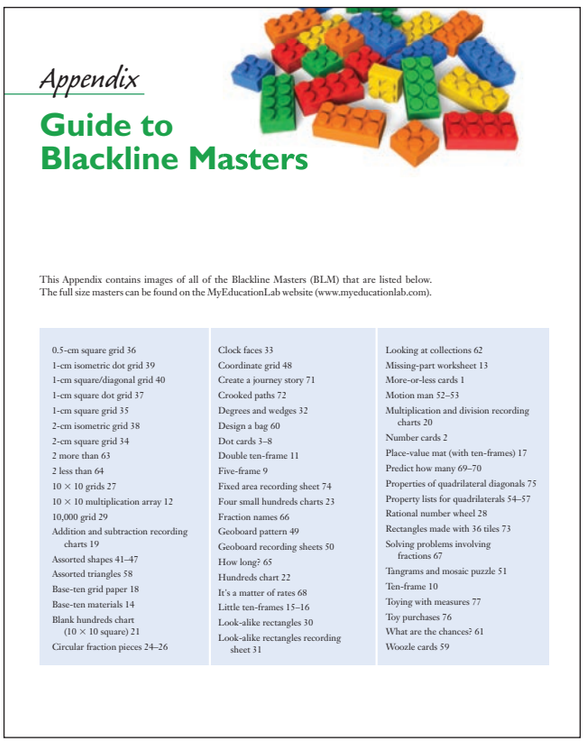
MyEducationLab™

Visit MyEducationLab to access an electronic version of the text, as well as a variety of topics that enhance the text material. The topics include the following to support your learning in the course:

- Assessment, including Building Teaching Skills and Dispositions and Video Assignments
- Discussion board questions
- Videos, simulations, activities, case studies, and other useful course resources

Appendix ▶

The Appendix contains images of all of the Blackline Masters.



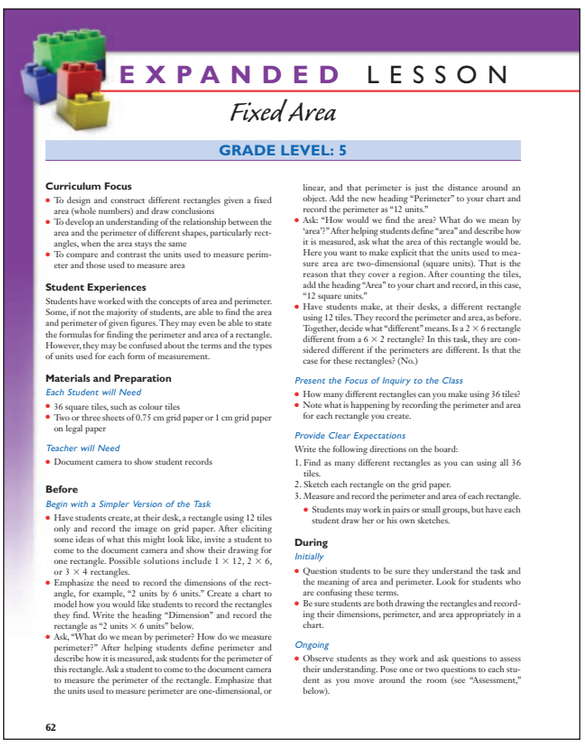
Appendix

Guide to Blackline Masters

This Appendix contains images of all of the Blackline Masters (BLM) that are listed below. The full size masters can be found on the MyEducationLab website (www.myeducationlab.com).

Expanded Lessons ▶

An example of an Expanded Lesson can be found at the end of Chapter 4. In addition, eight similar Expanded Lessons can be found on MyEducationLab at www.myeducationlab.com.



EXPANDED LESSON

Fixed Area

GRADE LEVEL: 5

Curriculum Focus

- To design and construct different rectangles given a fixed area (whole numbers) and draw conclusions
- To develop an understanding of the relationship between the area and the perimeter of different shapes, particularly rectangles, when the area stays the same
- To compare and contrast the units used to measure perimeter and those used to measure area

Student Experiences

Students have worked with the concepts of area and perimeter. Some, if not the majority of students, are able to find the area and perimeter of given figures. They may even be able to state the formulas for finding the perimeter and area of a rectangle. However, they may be confused about the terms and the types of units used for each form of measurement.

Materials and Preparation

Each Student will Need

- 36 square tiles, such as colour tiles
- Two or three sheets of 0.75 cm grid paper or 1 cm grid paper on legal paper

Teacher will Need

- Document camera to show student records

Before

Begin with a Simpler Version of the Task

- Have students create, at their desk, a rectangle using 12 tiles only and record the image on grid paper. After eliciting some ideas of what this might look like, invite a student to come to the document camera and show their drawing for one rectangle. Possible solutions include 1 × 12, 2 × 6, or 3 × 4 rectangles.
- Emphasize the need to record the dimensions of the rectangle, for example, "2 units by 6 units." Create a chart to model how you would like students to record the rectangles they find. Write the heading "Dimension" and record the rectangle as "2 units × 6 units" below.
- Ask, "What do we mean by perimeter? How do we measure perimeter?" After helping students define perimeter and describe how it is measured, ask students for the perimeter of this rectangle. Ask a student to come to the document camera to measure the perimeter of the rectangle. Emphasize that the units used to measure perimeter are one-dimensional, or linear, and that perimeter is just the distance around an object. Add the new heading "Perimeter" to your chart and record the perimeter as "12 units."
- Ask: "How would we find the area? What do we mean by 'area'?" After helping students define "area" and describe how it is measured, ask what the area of this rectangle would be. Here you want to make explicit that the units used to measure area are two-dimensional (square units). That is the reason that they cover a region. After counting the tiles, add the heading "Area" to your chart and record, in this case, "12 square units."
- Have students make, at their desks, a different rectangle using 12 tiles. They record the perimeter and area, as before. Together, decide what "different" means. Is a 2 × 6 rectangle different from a 6 × 2 rectangle? In this task, they are considered different if the perimeters are different. Is that the case for these rectangles? (No.)

Present the Focus of Inquiry to the Class

- How many different rectangles can you make using 36 tiles?
- Note what is happening by recording the perimeter and area for each rectangle you create.

Provide Clear Expectations

Write the following directions on the board:

1. Find as many different rectangles as you can using all 36 tiles.
2. Sketch each rectangle on the grid paper.
3. Measure and record the perimeter and area of each rectangle.
   - Students may work in pairs or small groups, but have each student draw her or his own sketches.

During

Initially

- Question students to be sure they understand the task and the meaning of area and perimeter. Look for students who are confusing these terms.
- Be sure students are both drawing the rectangles and recording their dimensions, perimeter, and area appropriately in a chart.

Ongoing

- Observe students as they work and ask questions to assess their understanding. Pose one or two questions to each student as you move around the room (see "Assessment," below).

After

- Bring the class together to share and discuss the task. (They should identify 5 different rectangles: 1 × 36, 2 × 18, 3 × 12, 4 × 9, and 6 × 6. If some students disagree that the 6 × 6 is a rectangle, prompt discussion on the properties of rectangles.)
- Ask students what they have found out about perimeter and area. Ask: "Did the perimeter stay the same? Is that what you expected? When is the perimeter big and when is it small?"
- Ask students how they can be sure they have all the possible rectangles.
- Ask students to describe what happens to the perimeter as the length and width change. (The perimeter gets shorter as the sides become similar in length. The square has the shortest perimeter.) Provide time to share ideas.

ASSESSMENT

Observe

- Are students confusing perimeter and area?
- As students form new rectangles, are they aware that the area is not changing because they are using the same number of tiles each time? These students may not know what area is, or they may be confusing it with perimeter.
- Are students looking for patterns for how to find the perimeter?
- Are students discussing important concepts or patterns with their partners?

Ask

- What is the area of the rectangle you just made?
- What is the perimeter of the rectangle you just made?
- How is area different from perimeter?
- How do you measure the area of a rectangle? The perimeter?

Supplements

MyEducationLab

MyEducationLab for Curriculum and Instruction (www.myeducationlab.com) is an online learning solution that provides interactive exercises designed to help teacher candidates develop the knowledge and skills that teachers need. Using classroom video, authentic student and teacher artifacts, and other resources and assessments, the learning experiences in MyEducationLab offer you a unique and valuable education tool.

For each mathematics topic covered, you will find the following features and resources:

- **Expanded Lesson Activities:** these expanded activities will help you explore how to use the lessons in the classroom.
- **Artifact Analysis Activities:** activities contain audio and visual records of students interacting with mathematics, accompanied by questions that focus on the analysis of student thinking.
- **Activities for Developing Mathematics Content Knowledge:** designed to assess and develop *your* mathematics content understanding, activities focus on areas where people typically have misconceptions.
- **Building Teaching Skills and Dispositions:** learning units that help teacher candidates practise and strengthen skills that are essential to effective teaching.
- **Video Assignments:** a robust selection of classroom videos accompanied by a set of questions.
- **IRIS Modules and Case Studies:** course-enhancement materials from the IRIS Center, designed to better prepare school personnel to provide an appropriate education to students.
- **Lesson Plan Builder:** an easy-to-use tool you can use to create, update, and share quality lesson plans.

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Instructor's Manual The **Instructor's Manual** includes a wealth of resources designed to help instructors teach the course, including chapter notes, activity suggestions, suggested assessments, and test questions.

PowerPoint Presentation Ideal for instructors to use for lecture presentations or student handouts, the PowerPoint presentation provides dozens of ready-to-use graphic and text images tied to the text.

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Acknowledgments from the US Edition

Many talented people have contributed to the success of this book, and we are deeply grateful to all those who have assisted over the years. Without the success of the first edition, there would certainly not have been a second, much less eight editions. John worked closely with Warren Crown, John Dossey, Bob Gilbert, and Steven Willoughby, who gave time and great care in offering detailed comments on the original manuscript.

In preparing the eighth US edition, we have received thoughtful input from the following educators who offered comments on the seventh edition or on the manuscript for the eighth:

Margaret Adams, University of North Carolina at Charlotte
Joohi Lee, University of Texas at Arlington
Sandra J. Phifer, Metropolitan State College of Denver
Diana Piccolo, Missouri State University
Janet Lynne Tassell, Western Kentucky University

Each reviewer challenged us to think through important issues. Many specific suggestions have found their way into this book, and their feedback helped us focus on important ideas. We are indebted to these committed professionals.

We also would each like to thank our families for their many contributions and support. On behalf of John, we thank his wife of more than 40 years, Sharon. Sharon was John's biggest supporter in this process and remained a sounding board for his many decisions as he wrote the first six editions of this book. We also thank his daughters, Bridget (a fifth-grade teacher in Chesterfield County, Virginia) and Gretchen (an associate professor of psychology at Rutgers University–Newark). They were John's first students and he tested many ideas that are in this book by their sides. We can't forget those who called John "Math Grandpa": his granddaughters, Maggie, Aidan, and Gracie.

From Karen Karp: I would like to express thanks to my husband, Bob Ronau, who as a mathematics educator graciously helped me think about decisions while offering insights and encouragement. In addition, I thank my children, Matthew, Tammy, Joshua, Misty, Matt, Christine, Jeffrey, and Pamela for their kind support and inspiration. I also am grateful for my wonderful grandchildren, Jessica, Zane, and Madeline, who have helped deepen my understanding about how children think.

From Jennifer Bay-Williams: I am so grateful to my husband, Mitch, who offers support, guidance, and wisdom to my writing, and my children, MacKenna (8 years) and Nicolas (6 years), who enjoy doing a little extra math from time to time. My parents, siblings, and nieces and nephews have all contributed ideas and support to the writing of this edition. Finally, I want to thank Brandy Jones, who has been invaluable in helping me find research to inform my writing for this edition.

Most importantly, we thank all the teachers and students who gave of themselves by assessing what worked and what didn't work in the many iterations of this book. In particular for the eighth edition, we thank teachers who generously tested activities and provided student work for us: Kyle Patterson, Kim George, Kelly Eaton, Sarah Bush, and Elizabeth Popelka. If future teachers learn how to teach mathematics from this book, it is because teachers and children before them shared their best ideas and thinking with the authors.

Acknowledgements from the Canadian Edition

From Lynn McGarvey: I am indebted to my husband, Don, and my son, Matthew, who have been incredibly patient and supportive throughout the many phases of this project. I need to thank the creative and inspiring instructors who have been part of the Mathematics Subject Area Committee at the University of Alberta. These educators are constantly expanding my understanding of mathematics teaching and learning: Jane Gannon, Audrey Hodgson-Ward, Bev Kula, Julie Long, Janelle McFeetors, Gladys Sterenberg, and Carrie Watt. Finally, my contribution to the textbook has been with my students in mind. They have been instrumental in challenging me to connect theory, practice, and their personal histories to what it means to think mathematically.

In preparing this fourth Canadian edition, the following educators were among those who offered helpful and insightful comments on the previous edition and the draft manuscript:

Jerry Ameis, University of Winnipeg
Jayne Asselin, Fleming College
David Bale, University of Regina
Norma Fraser, Thompson Rivers University
Nahid Golafshani, Laurentian University
Audrey Hodgson-Ward, University of Alberta
Harpreet Kaur, Simon Fraser University
Wendy Klassen, UBC Okanagan
Jo-Anne Lake, Queen's University
Annie Savard, McGill University
Gladys Sterenberg, University of Alberta
David Watson, Western University